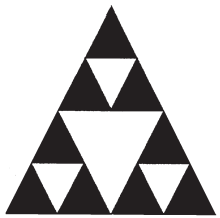
ANSWERS

I. 1. a) See videotape for details of the geometric approach and the Chaos game.

b) Second iteration of the *ST:*



2.

a) $x_0 = 1$; $x_1 = 3$; $x_2 = 9$; $x_3 = 27$; $x_4 = 81$

b) $x_n = 3$ or $f(n) = 3$

c) $x_n = 3x_{n-1}$, with $x_0 = 1$ or $f(n = 3f(n-1)$, with $f(0) = 1$.

3.

a) A_5=$7,346.64;
A_{10}=$10,794.62

b) A_n=5,000(1.08)^n or
f(n) =5,000(1.08)^n

c) $A_n = (1.08)A_{n-1}$, with $A_0 = 5{,}000$ or
$f(n) = (1.08)f(n-1)$, with $f(0) = 5{,}000$

4.

a) *$421.32.* The fact that there is still an outstanding balance of *$421.32* implies that a monthly payment of *$350.00* is not large enough.

b) $r = 1$ or $r = -1$ mean that the regression line is a perfect fit. The negative sign signifies a decreasing function.

c) The monthly payment. The final balance after *4* years or *48* months.

d) *(357.40, 0)* mean that a monthly payment of *$357.40* will pay off the loan in *4* years, leaving a final, out standing balance of *$0.00.*
(0, 20,347.34) means that a monthly payment of *$0.00* will yield a final, outstanding balance of *$20,347.34* after *4* years.

e) *$17,655.20.* (A down payment of *$500.00* plus *48* payments of *$357.40.*)

II. 1. a) Each existing square is divided into nine equal squares by a system of four line segments, just like a tic-tac-toe game. Of these nine, newly formed squares, five are preserved (the *4* corner-squares and the central one) and four are discarded to yield the next iteration.

b) 625

c) $x_n = 5^n$ or
$f(n) = 5^n$ (Explicit)

$x_n = 5x_{n-1}$, with $x_0 = 1$, or
$f(n) = 5f(n-1)$, with $f(0) = 1$ (Recursive)

III. 1. a) *$4,359.88. $6;336.19*

b) $A_n = (1 + .075/12)A_{n-1} + 100$, with $A_0 = 3{,}000$. [Calculator: Enter *3,000* and iterate *(1+.075/12)ANS+100.*]

c) $A_{60} = \$11{,}612.59$ (After *5* years)
$A_{120} = \$24{,}129.29$ (After *10* years)

d) *11* years and *10* months. (You will have invested a total of *$17,200:* The initial principal of *$3,000* plus *142* payments of *$100.*)

ALGEBRA 1

The Complete Course

Lesson Thirty

Section VII:
Iterating Functions

Using Iteration as a Problem-Solving Tool

KA8460

Worksheet

HOW TO USE THE VIDEO AND WORKSHEET

1. The "STOP TO THINK" signal means pause to think.
2. The "STOP TO WORK" signal means work the problem(s).
3. Rewind the tape and watch the lesson again if the concept is not clear.
4. Students should complete the exercises on the worksheet to confirm their understanding of this lesson.

I. VIDEOTAPE FOLLOW-UP QUESTIONS

1. In Exploration 1 we obtained the Sierpinski Triangle (*ST*) in two different ways.

   a) Briefly explain the two approaches.

   b) Draw the second iteration of the *ST,* using the geometric approach.

2. Consider the geometric approach to the *ST:*

   a) Give the first five elements, x_0 through x_4, of the sequence x_n, where x_n denotes the number of triangles remaining in the *ST* after *n* iterations.

   b) Give the explicit formula (or rule) for x_n. (Express your answer in both sequence and function notation.)

   c) Define x_n recursively. (Again, give your answer in both notations.)

3. In Exploration 2 we explored the concept of simple annual interest. Suppose you invested *$5,000* at *8%* simple annual interest instead of *$1,000* at *6%*.

   a) Let A_n be the amount of money in your bank after *n* years. Compute A_{10} and A_{20}.

   b) Define A_n explicitly. (Express your answer in both sequence and function notation.)

   c) Define A_n recursively. (Again, give your answers in both notations.)

4. In Exploration 3, in order to compute the final balance after *4* years, we iterated the following recurrence relation *48* times on our graphing calculator:

   ANS (1+.085/12)-X, where *14,500* was the initial value and *X* the monthly payment.

   a) Compute the final balance after *4* years for *X=350.* What is the significance of your answer?

   b) We found *y=-56.93x+20,347.34* to be the regression line for this problem, and *r=-1* the correlation coefficient. Explain the meaning of *r=-1.*

   c) What variable is measured on the x-axis (the input axis)? What variable is measured on the *y*-axis (the output axis).

   d) What is the meaning of the *x*-intercept we found, *(357.40, 0)?* What is the meaning of the *y*-intercept we found, *(0, 20,347.34)?*

   e) How much money did you end up paying for your $15,000-dollar car?

II. SUPPLEMENTARY EXERCISES

1. The following is the geometric construction of another fractal called the "Box Fractal" (*BF*):

n=0 n=l (1^{st} iteration) n=2 (2^{nd} iteration)

As with the triangles in the *ST,* the squares in the *BF* increase in number and decrease in size, as the number of iterations increases.

a) Describe the geometric iterative process that generates the *BF.*

b) How many squares are there in the 4^{th} iteration of the *BF.*

c) Let x_n denote the number of squares in the *BF* after *n* iterations. Define x_n explicitly and recursively. (Use the notation of your choice.)

III. INVESTIGATIVE PROBLEM

1. Suppose you invest *$3,000* in a bank that pays *7.5%* interest on your money, compounded monthly

   a) How much money would you have in your account after 5 years? After *10* years?

   b) Now suppose that each month you deposit $100 in your account, in addition to the interest that accrues. Write a recurrence relation for A_n, the amount of money in the bank after *n* months.

   c) Compute A_{60} and A_{120} using the iteration feature on your graphing calculator.

   d) Continuing to add *$100* to your account each month, how long would you have to wait for your investment to be worth *$30,000* (or *10* times the initial principal)?