ANSWERS

I.

1.

a) *1.*

b) *x*
x + 4
3(x+4) = 3x + 12 (The distributive law - See Lesson 3.)
3x + 12 - 9 = 3x + 3
2(3x+3) = 6x + 6
(6x+6)/6 = (6x)/6 + 6/6 = x + 1
x + 1 - x = 1

2. Answers will vary.

3. Example:

7	*8*	*9*
14	*15*	*16*
21	*22*	*23*

a) The sum of the *9* numbers is *135.*

b) *135/15=9*

c) Let n be the central number. We can write the 3x3 array as follows:

n-8	*n-7*	*n-6*
n-1	*n*	*n+1*
n+6	*n+7*	*n+8*

The sum of the 9 numbers is: *(n-8)+(n-7)+(n-6) +(n-1)+n(n+1)+(n+6)+(n+7)+(n+8)= [(n-8)+(n+8)]+[(n+7)+(n-7)+[(n-6)+(n+6)]+[(n-1)+(n+1)]+n=2n+2n+2n+2n+n=9n.*

Dividing 9*n* by *n* gives *9*!

II.

1.

a) *x.*

b) Yes.

c) *x*
x+1
9(x+1) = 9x + 9
9x + 9 + x = l0x + 9
10x + 9 - 4 = l0x + 5

Deleting the ones digit means deleting the digit *5*. We are left with "*10x*" which means that the original number has "moved" to the tens place.

2.

a) $\frac{1}{x}+\frac{1}{y}=\frac{y+x}{xy}$

b) $\frac{x+y}{w+z}=\frac{x}{w+z}+\frac{y}{w+z}$

c) $(a+b)^2 = a^2 + ab + b^2$ (See Lesson 3)

d) $x^2x^3 = x^5$

e) $a^{-1} = 1/a$

f) $(-x) + (-x) = -2x$

III.

1. The error arises when both sides of the equation are divided by *(x-y):*

This factor is zero since x=y, and dividing by 0 is an error because it gives an undefined (a nonsensical) quotient.

ALGEBRA 1

The Complete Course

Lesson One

Section I:
Exploring Algebra

An Overview

KA8431

Worksheet

HOW TO USE THE VIDEO AND WORKSHEET

1. The "STOP TO THINK" signal means pause to think.
2. The "STOP TO WORK" signal means work the problem(s).
3. Rewind the tape and watch the lesson again if the concept is not clear.
4. Students should complete the exercises on the worksheet to confirm their understanding of this lesson.

I. VIDEOTAPE FOLLOW-UP QUESTIONS

1. The statement of the Think-of-a-number problem we saw at the end of the lesson went as follows:

   Think of a number.
   Add *4*.
   Multiply by *3*.
   Subtract *9*.
   Multiply by *2*.
   Divide by *6*.
   Subtract the original number.

   a) What did you obtain?

   b) Can you show that no matter what the starting number *x*, the answer will always be *1*?

2. Make up a similar problem that will give *5* no matter what the starting value.

3. Select a 3x3 array of numbers from a calendar (any month, any year).

   a) Add up the nine numbers.

   b) Divide the sum by the central number. What do you obtain?

   c) Can you show that no matter what the 3x3 array of numbers, the answer will always be *9*?

II. SUPPLEMENTARY EXERCISES

1.

   Think of a number.
   Add *1*.
   Multiply by *9*.
   Add the original number.
   Subtract *4*.
   Delete the ones digit.

   a) What do you obtain?

   b) Do you think that you will always obtain the original number?

   c) Can you prove your answer?

2. The following are some common errors students make in algebra. Correct the following errors by writing a true equation in each case:

   a) $\frac{1}{x}+\frac{1}{y}=\frac{2}{x+y}$

   b) $\frac{x+y}{w+z}=\frac{x}{w}+\frac{y}{z}$

   c) $(a+b)^2 = a^2 + b^2$

   d) $x^2 \bullet x^3 = x^6$

   e) $a^{-1} = -a^1$

   f) $(-x) + (-x) = +2x$

III. INVESTIGATIVE PROBLEM

1. The paradox illustrates how fallacies can arise in algebra when algebraic operations are applied incorrectly. Find the source of the error.

Let $x = y$	:	$x = y$
Multiply both sides by x	:	$x^2 = xy$
Subtract y^2 from both sides	:	$x^2 - y^2 = xy - y^2$
Factor*	:	$(x + y)(x - y) = y(x - y)$
Divide both side by $(x-y)$	:	$x + y = y$
Since $x=y$, substitute y for x	:	$2y = y$
Divide both sides by y	:	$2 = 1$

* You will see in Lesson 3 that $x^2 - y^2 = (x+y)(x-y)$